Technical Appendix to: Austerity in the Aftermath of the Great Recession*

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1 More Details on the Economy with Incomplete Markets

1.1 Household Budget Constraint

The nominal budget constraints for the representative household in country n are

$$P_{n,t}\left[(1+\tau^C_{n,t})C_{n,t}+X_{n,t}\right]+(1-\delta)\,\mu_{n,t}K_{n,t}+B_{n,t}+\sum_{j=1}^{n}\frac{E_{j,t}S^j_{n,t}}{E_{n,t}}+\mathbb{I}_{\text{comp}}\left[\sum_{s^{t+1}}\frac{a\left(s^t,s_{t+1}\right)b_n(s^t,s_{t+1})}{E_{n,t}}-\frac{b_n(s^{t-1},s_t)}{E_{n,t}}\right]$$
$$=\mu_{n,t}K_{n,t+1}+(1-\tau^L_{n,t})W_{n,t}L_{n,t}+\Pi^f_{n,t}+\Pi^e_{n,t}+(1+i_{t-1})F(\lambda_{n,t-1})e^{\epsilon^F_{n,t-1}}B_{n,t-1}+\sum_{j=1}^{n}\frac{E_{j,t}\left(1+i_{j,t-1}\right)S^j_{n,t-1}}{E_{n,t}}-T_{n,t},$$

where $B_{n,t}$ are loans extended to entrepreneurs. Let us consider the case with incomplete markets, so that the term $\mathbb{I}_{\text{comp}}=0$. Let country n's net foreign asset position denoted in the reserve currency at time t be $S^*_{n,t}$, so that $S^n_{n,t}+\frac{S^*_{n,t}}{E_{n,t}}=S_{n,t}$, where $S_{n,t}$ are country n's total bond holdings valued in domestic currency (see below). Then,

$$P_{n,t}\left[(1+\tau^C_{n,t})C_{n,t}+X_{n,t}\right]+(1-\delta)\,\mu_{n,t}K_{n,t}+B_{n,t}+S^n_{n,t}+\frac{S^*_{n,t}}{E_{n,t}}=\mu_{n,t}K_{n,t+1}+(1-\tau^L_{n,t})W_{n,t}L_{n,t}$$
$$+\Pi^f_{n,t}+\Pi^e_{n,t}+(1+i_{t-1})F(\lambda_{n,t-1})e^{\epsilon^F_{n,t-1}}B_{n,t-1}+\frac{\left(1+i^*_{t-1}\right)S^*_{n,t-1}}{E_{n,t}}+(1+i_{n,t-1})S^n_{n,t-1}-T_{n,t}.$$

Inserting the government budget constraint

$$P_{n,t}G_{n,t}=T_{n,t}+\tau^C_{n,t}P_{n,t}C_{n,t}+\tau^L_{n,t}W_{n,t}L_{n,t}+\tau^K_{n,t}u_{n,t}R_{n,t}K_{n,t}.$$

gives

$$P_{n,t}\left(C_{n,t}+X_{n,t}+G_{n,t}\right)+(1-\delta)\,\mu_{n,t}K_{n,t}+B_{n,t}+S^n_{n,t}+\frac{S^*_{n,t}}{E_{n,t}}=\mu_{n,t}K_{n,t+1}+W_{n,t}L_{n,t}+\Pi^f_{n,t}+\Pi^e_{n,t}$$
$$+(1+i_{t-1})F(\lambda_{n,t-1})e^{\epsilon^F_{n,t-1}}B_{n,t-1}+\frac{\left(1+i^*_{t-1}\right)S^*_{n,t-1}}{E_{n,t}}+(1+i_{n,t-1})S^n_{n,t-1}+\tau^K_{n,t}u_{n,t}R_{n,t}K_{n,t}.$$

Note that $C_{n,t}+X_{n,t}+G_{n,t}=Y_{n,t}-a(u_{n,t})K_{n,t}$ and profits are defined as

$$W_{n,t}L_{n,t}+\Pi^f_{n,t}=p_{n,t}Q_{n,t}-R_{n,t}u_{n,t}K_{n,t},$$

so that the budget constraint simplifies to

$$
\begin{aligned}
&P_{n,t}\left(Y_{n,t}-a(u_{n,t})K_{n,t}\right)+(1-\tau^K_{n,t})R_{n,t}u_{n,t}K_{n,t}+(1-\delta)\,\mu_{n,t}K_{n,t}+B_{n,t}+S^n_{n,t}+\frac{S^*_{n,t}}{E_{n,t}}\\
&=\mu_{n,t}K_{n,t+1}+p_{n,t}Q_{n,t}+\Pi^e_{n,t}+(1+i_{t-1})F(\lambda_{n,t-1})e^{\epsilon^F_{n,t-1}}B_{n,t-1}+\frac{\left(1+i^*_{t-1}\right)S^*_{n,t-1}}{E_{n,t}}+(1+i_{n,t-1})S^n_{n,t-1}.
\end{aligned}
$$

To simplify this expression further, we set up the budget constraint for entrepreneurs:

$$
\begin{aligned}
&\mu_{n,t}K_{n,t+1}+\Pi^e_{n,t}+(1+i_{t-1})F(\lambda_{n,t-1})e^{\epsilon^F_{n,t-1}}B_{n,t-1}\\
&=(1-\tau^K_{n,t})R_{n,t}u_{n,t}K_{n,t}-P_{n,t}a(u_{n,t})K_{n,t}+(1-\delta)\mu_{n,t}K_{n,t}+B_{n,t}.
\end{aligned}
$$

Inserting this into the household budget constraint gives

$$
P_{n,t}Y_{n,t}+S^n_{n,t}+\frac{S^*_{n,t}}{E_{n,t}}=p_{n,t}Q_{n,t}+\frac{\left(1+i^*_{t-1}\right)S^*_{n,t-1}}{E_{n,t}}+(1+i_{n,t-1})S^n_{n,t-1}.
$$

In equilibrium, $S^n_{n,t}=0$, so that

$$
P_{n,t}Y_{n,t}+\frac{S^*_{n,t}}{E_{n,t}}=p_{n,t}Q_{n,t}+\frac{\left(1+i^*_{t-1}\right)S^*_{n,t-1}}{E_{n,t}}.
$$

1.2 Steady-State Bond Holdings

In steady state, $P_n=p_n=E_n=1$. Using $NX_n=Q_n-Y_n$, we have

$$
\begin{aligned}
Y_n+S^*_n&=Q_n+\frac{S^*_n}{\beta}\\
\frac{S^*_n}{Y_n}&=\frac{\beta}{\beta-1}\frac{NX_n}{Y_n}.
\end{aligned}
$$

1.3 Uncovered Interest Rate Parity

To analyze dynamics in a country's net foreign asset position, we would need to know the currency composition of that net foreign asset position because the currency composition determines how exchange rates fluctuatuations translate into valuation effects. We lack data on the currency composition of countries' net foreign asset position and therefore assume that net foreign assets are proportional to a country's net export position. That is, we assume a clearing house For example, if France's overall net exports are -6% of its domestic absorption, but they are 2% vis-a-vis the US and -10% vis-a-vis Germany; then France's net foreign assets are composed of $\frac{\beta}{\beta-1}$*2% of US (USD) bonds and $-\frac{\beta}{\beta-1}$*10% of German (Euro) bonds.[1] Let country n's total net foreign asset position denoted at time t be $S^*_{n,t}$, so that $S^n_{n,t} + \frac{S^*_{n,t}}{E^*_{n,t}} = S_{n,t}$, where $S_{n,t}$ are country n's total bond holdings valued in domestic currency, and $E^*_{n,t}$ is country n's effective exchange rate weighted by its bilateral net export positions.

We now also explicitly introduce a penalty term that represents a small costs on holding claims on other countries. This ensures that the resulting equilibrium is stationary. We model this cost using a quadratic function $\frac{\iota}{2Y^*_n}\left(S^*_n - S^*_{n,t}\right)^2$.

Then, the budget constraints for the representative household in country n are

$$P_{n,t}Y_{n,t} + \sum_j \frac{E_{j,t}S^j_{n,t}}{E_{n,t}} + \frac{\iota}{2Y^*_n}\left(S^*_n - S^*_{n,t}\right)^2 = p_{n,t}Q_{n,t} + \sum_j \frac{\left(1+i_{j,t-1}\right)S^j_{n,t-1}}{E^j_{n,t}}.$$

The Euler equations associated with the non-contingent nominal bonds $S_{n,t}$ and $S^j_{n,t}$ require

$$\lambda_{n,t} = (1+i_{n,t})\sum_{s^{t+1}} \pi(s^{t+1}|s^t)\beta\lambda_{n,t+1}$$

and

$$\lambda_{n,t}\left(\frac{E_{j,t}}{E_{n,t}} - \iota\frac{S^*_n - S^*_{n,t}}{Y^*_n}\right) = \beta(1+i^j_t)\sum_{s^{t+1}} \pi(s^{t+1}|s^t)\lambda_{n,t+1}\frac{E_{j,t+1}}{E^*_{n,t+1}},$$

where here, λ denotes the Lagrange multiplier on the budget constraint (not the leverage ratio of the entrepreneurs). The log-linearized equations are

$$\tilde{\lambda}_{n,t} = \beta\Delta i_{n,t} + \tilde{\lambda}_{n,t+1}$$
$$\tilde{\lambda}_{n,t} + \tilde{E}_{j,t} - \tilde{E}_{n,t} + \iota\frac{S^*_n}{Y_n}\tilde{S}^*_{n,t} = \beta\Delta i_{j,t} + \tilde{\lambda}_{n,t+1} + \tilde{E}_{j,t+1} - \tilde{E}_{n,t+1}.$$

[1] If net exports are zero in steady state, then net foreign assets do not move up to a first-order approximation.

Inserting the domestic Euler equation into the international Euler equation to replace $\tilde{\lambda}_{n,t}$:

$$\widetilde{\Delta E}_{n,t+1} - \widetilde{\Delta E}_{j,t+1} + \iota \frac{S_n^*}{Y_n} \tilde{S}_{n,t}^* = \beta(\Delta i_{j,t} - \Delta i_{n,t}),$$

where $\widetilde{\Delta E}_{n,t+1} = \tilde{E}_{n,t+1} - \tilde{E}_{n,t}$. This uncovered interest rate parity can be rewritten

$$\beta \Delta i_{n,t} - \tilde{\pi}_{n,t+1} + \tilde{e}_{n,t+1} - \tilde{e}_{n,t} + \iota \frac{S_n^*}{Y_n} \tilde{S}_{n,t}^* = \beta \Delta i_{j,t} - \tilde{\pi}_{j,t+1} + \tilde{e}_{j,t+1} - \tilde{e}_{j,t}.$$

Note that we normalize $e_{1,t} = 1$ for all periods.

2 Steady State

We solve the model in a neighborhood of a non-stochastic steady state with zero inflation. Because inflation is zero, the Euler equations associated with the uncontingent nominal bonds imply that the nominal interest rate is $1 + i_n = \frac{1}{\beta}$ for all n. Next, we use the entrepreneurs' first-order condition for capital,

$$(1 + i_n)F(\lambda_n) = \frac{(1 - \tau_n^K)u_n R_n + \mu_n \left(1 - \delta(1 - \tau_n^K)\right) - P_n a\left(u_n\right)}{\mu_n}.$$

Note that the households' first-order condition for investment,

$$\frac{U_{1,n}}{1 + \tau_n^C} = \frac{\mu_n}{P_n} \frac{U_{1,n}}{1 + \tau_n^C}\left(1 - f - f'\right) + \beta \left[\frac{\mu_n}{P_n} \frac{U_{1,n}}{1 + \tau_n^C} f'\right]$$

implies that $\mu_n = P_n$ because $f = f' = 0$ in steady state. Inserting this back into the entrepreneurs' first-order condition for capital and noting that $a(u_n) = 0$ and $u_n = 1$ gives

$$\begin{aligned} \frac{F_n}{\beta} &= (1 - \tau_n^K)\left(r_n + 1 - \delta\right) \\ r_n &= \frac{1}{1 - \tau_n^K}\left(\frac{F_n}{\beta} - 1\right) + \delta, \end{aligned} \tag{2.1}$$

where we have defined the steady state interest rate spreads $F_n \equiv F_n\left(\lambda\right)$. Below we calibrate these spreads to match their observable counterparts. Once we have calibrated F_n, the equation above determines the real rental price of capital $r_n \equiv R_n/P_n$ in each country.

With zero inflation, the steady state price of intermediates is a constant markup over the nominal marginal cost,

$$p_n = \frac{\psi_q}{\psi_q - 1} MC_n.$$

This can be seen from the reset equation and the law of motion for the nominal price of the intermediate good.

Next, cost minimization of the first-stage producers implies

$$
\begin{aligned}
R_n &= MC_n \alpha Z_n \left[\frac{K_n}{L_n}\right]^{\alpha-1} \\
r_n &= \frac{\psi_q - 1}{\psi_q} \frac{p_n}{P_n} \alpha Z_n \left[\frac{K_n}{L_n}\right]^{\alpha-1} \\
\frac{p_n}{P_n} &= r_n \frac{\psi_q}{\psi_q - 1} \frac{1}{\alpha Z_n} \left[\frac{K_n}{L_n}\right]^{1-\alpha}
\end{aligned}
$$

We adjust the technology levels Z_n so that all intermediate goods prices equal the price of the respective final good: $p_n = P_n$.

Then, the price index formula for the final good states

$$
\begin{aligned}
P_n &= \left(\sum_{j=1}^{N} \omega_n^j \left[\frac{E_j}{E_n} p_j\right]^{1-\psi_y}\right)^{\frac{1}{1-\psi_y}} \\
P_n E_n &= \left(\sum_{j=1}^{N} \omega_n^j \left[P_j E_j \frac{p_j}{P_j}\right]^{1-\psi_y}\right)^{\frac{1}{1-\psi_y}}
\end{aligned}
$$

One can easily verify that $P_n E_n = 1$ solves this equation, that is the real exchange rate $e_n = P_n E_n$ is unity.[2]

We directly calibrate some steady-state variables to match their empirical counterparts. Those are the shares of government purchases, G_n, the relative country sizes, $\frac{\mathbb{N}_j Y_j}{\mathbb{N}_n Y_n}$ and the bilateral import shares $\frac{y_n^j}{Y_n}$. We now derive the shares of the remaining variables, NX_n, C_n and X_n.

To derive the share of net exports, we first use the demand equation for intermediate goods,

$$
\begin{aligned}
y_n^j &= Y_n \omega_n^j \left[\frac{E_j}{E_n} \frac{p_j}{P_n}\right]^{-\psi_y} \\
&= Y_n \omega_n^j \left[\frac{e_j}{e_n} \frac{p_j}{P_j}\right]^{-\psi_y}.
\end{aligned}
$$

It follows that ω_n^j is country n's import share of country j's good, measured in terms of the

[2] We can also set $e_n = \frac{1}{\varrho}$ for any constant $\varrho > 0$.

final good Y_n:

$$\omega_n^j = \frac{y_n^j}{Y_n}.$$

It is also useful to define import shares in terms of *domestic absorption*, $P_n Y_{n,T} = P_n Y_n + \upsilon_n p_n G_n$:[3]

$$\omega_{n,T}^j = \frac{y_n^j}{Y_{n,T}} \quad \forall j \neq n \qquad \text{and} \qquad \omega_{n,T}^n = \frac{y_n^n}{Y_{n,T}} + \frac{\upsilon_n G_n}{Y_{n,T}}.$$

The implied net export share can then be expressed in terms of country sizes and the import preference parameters. Inserting the market clearing condition for Q_n into the definition of net exports, $NX_n = p_n Q_n - P_n Y_{n,T}$, we have[4]

$$\frac{NX_n}{P_n Y_{n,T}} = \left(\sum_{j=1}^{N} \frac{\mathbb{N}_j Y_{j,T}}{\mathbb{N}_n Y_{n,T}} \omega_{j,T}^n \right) - 1 \tag{2.2}$$

To derive the share of investment, we insert the marginal product of capital equation, $p_n Q_n = \frac{\psi_q}{\psi_q - 1} \frac{R_n}{\alpha} K_n$, into the definition of net exports, $NX_n = p_n Q_n - P_n Y_{n,T}$:

$$\begin{aligned} \frac{\psi_q}{\psi_q - 1} \frac{R_n}{\alpha\delta} X_n &= P_n Y_{n,T} + NX_n \\ \frac{X_n}{Y_{n,T}} &= \frac{\alpha\delta}{\frac{\psi_q}{\psi_q - 1} r_n} \left(1 + \frac{NX_n}{P_n Y_{n,T}} \right), \end{aligned} \tag{2.3}$$

[3] Remember that $P_n = p_n$ in steady state.

[4]

$$\begin{aligned} NX_n &= p_n \left(\sum_{j=1}^{N} \frac{\mathbb{N}_j}{\mathbb{N}_n} y_j^n \right) - p_n \upsilon_n G_n - P_n Y_{n,T} \\ \frac{NX_n}{P_n Y_{n,T}} &= \left(\sum_{j=1}^{N} \frac{\mathbb{N}_j p_n y_j^n}{\mathbb{N}_n P_n Y_{n,T}} \right) + \frac{p_n \upsilon_n G_n}{P_n Y_{n,T}} - 1 \\ &= \left(\sum_{j=1}^{N} \frac{\mathbb{N}_j Y_{j,T}}{\mathbb{N}_n Y_{n,T}} \frac{y_j^n}{Y_{j,T}} \right) + \frac{\upsilon_n G_n}{Y_{n,T}} - 1 \\ &= \left(\sum_{j=1}^{N} \frac{\mathbb{N}_j Y_{j,T}}{\mathbb{N}_n Y_{n,T}} \omega_{j,T}^n \right) - 1 \end{aligned}$$

where $X_n = \delta K_n$.

Finally, the consumption share is the residual of the market clearing condition $Y_{n,T} = C_n + X_n + G_n$:

$$\frac{C_n}{Y_{n,T}} = 1 - \frac{X_n}{Y_{n,T}} - \frac{G_n}{Y_{n,T}}. \tag{2.4}$$

To summarize, we solve for the steady state values as follows:

1. Calibrate the tax rate τ_n^K, the risk premium F_n and the government expenditure share $\frac{G_n}{Y_{n,T}}$ to their counterparts in the data.

2. Solve for the real rental price r_n using equation (2.1).

3. Calibrate the import preference parameters $\omega_{n,T}^j$ using data on country j's share of country n's imports, and calibrate the relative size of countries in terms of their domestic absorption, $\frac{\mathbb{N}_j Y_{j,T}}{\mathbb{N}_n Y_{n,T}}$.

4. Solve for the net export share $\frac{NX_n}{Y_{n,T}}$ using equation (2.2), the investment share $\frac{X_n}{Y_{n,T}}$ using equation (2.3) and the consumption share $\frac{C_n}{Y_{n,T}}$ using equation (2.4)

5. Solve for the parameters ω_n^j and υ_n using data on bilateral trade data on total trade and data on the import share of G relative to the total import share, $m_n^G \equiv (1 - \omega_{n,G}^n)/(1 - \hat{\omega}_{n,T}^n)$:[5]

$$\upsilon_n = \frac{1 - m_n^G}{1 - m_n^G \frac{G_n}{Y_{n,T}}}$$
$$\omega_n^n = 1 - \frac{1 - \omega_{n,T}^n}{1 - \upsilon_n \frac{G_n}{Y_{n,T}}}$$
$$\omega_n^j = \omega_{n,T}^j \frac{1 - \omega_n^n}{1 - \omega_{n,T}^n} \qquad \forall j \neq n$$

[5]First, note that

$$\omega_{n,G}^n = \upsilon_n + (1 - \upsilon_n)\omega_n^n$$
$$\omega_n^n = \frac{\omega_{n,G}^n - \upsilon_n}{1 - \upsilon_n}$$

3 Log-Linearized Equilibrium Conditions

3.1 Equilibrium Conditions

1. Domestic Euler equation

$$\frac{U_{1,n,t}}{(1+\tau^C_{n,t})P_{n,t}} = (1+i_{n,t})\sum_{s^{t+1}}\pi(s^{t+1}|s^t)\beta\frac{U_{1,n,t+1}}{(1+\tau^C_{n,t+1})P_{n,t+1}}$$

$$\beta\Delta i_{n,t} - \tilde{\pi}_{n,t+1} = \tilde{U}_{1,n,t} - \tilde{U}_{1,n,t+1} - \frac{\Delta\tau^C_{n,t} - \Delta\tau^C_{n,t+1}}{1+\tau_n}$$

Then,

$$\begin{aligned}
\omega^n_{n,T} &= \omega^n_n\frac{Y_n}{Y_{n,T}} + \frac{\upsilon_n G_n}{Y_{n,T}} \\
\omega^n_{n,T} &= \frac{\omega^n_{n,G}-\upsilon_n}{1-\upsilon_n}\frac{Y_{n,T}-\upsilon_n G_n}{Y_{n,T}} + \frac{\upsilon_n G_n}{Y_{n,T}} \\
&= \frac{\omega^n_{n,G}-\upsilon_n}{1-\upsilon_n} + \left(1-\frac{\omega^n_{n,G}-\upsilon_n}{1-\upsilon_n}\right)\frac{\upsilon_n G_n}{Y_{n,T}} \\
(1-\upsilon_n)\omega^n_{n,T} &= \omega^n_{n,G}-\upsilon_n + \left(1-\omega^n_{n,G}\right)\frac{\upsilon_n G_n}{Y_{n,T}} \\
\left[1-\omega^n_{n,T} - \left(1-\omega^n_{n,G}\right)\frac{G_n}{Y_{n,T}}\right]\upsilon_n &= \omega^n_{n,G} - \omega^n_{n,T} \\
\left[1-\frac{1-\omega^n_{n,G}}{1-\omega^n_{n,T}}\frac{G_n}{Y_{n,T}}\right]\upsilon_n &= 1-\frac{1-\omega^n_{n,G}}{1-\omega^n_{n,T}} \\
\upsilon_n &= \frac{1-m^G_n}{1-m^G_n\frac{G_n}{Y_{n,T}}}
\end{aligned}$$

And then

$$\begin{aligned}
1-\omega^n_{n,G} &= (1-\upsilon_n)\left(1-\omega^n_n\right) \\
\omega^n_n &= 1-m^G_n\frac{1-\omega^n_{n,T}}{1-\upsilon_n} \\
\omega^n_n &= 1-\frac{1-\omega^n_{n,T}}{1-\upsilon_n\frac{G_n}{Y_{n,T}}}
\end{aligned}$$

2. Wage Phillips curve ($w_{n,t} \equiv \frac{W_{n,t}}{P_{n,t}}$) [6]

$$\theta_w \tilde{\pi}^w_{n,t} = (1-\theta_w)(1-\theta_w\beta)\left[\widetilde{U}_{2,n,t} - \widetilde{U}_{1,n,t} + \frac{\Delta\tau^L_{n,t}}{1-\tau^L_n} + \frac{\Delta\tau^C_{n,t}}{1+\tau^C_n} - \widetilde{w}_{n,t}\right] + \theta_w\beta\tilde{\pi}^w_{n,t+1}$$

[6]The reset equation and law of motion for the nominal price in log-linearized form:

$$\widetilde{W}_{n,t} = \theta_w \widetilde{W}_{n,t-1} + (1-\theta_w)\tilde{w}^*_{n,t}$$
$$\widetilde{w}^*_{n,t} = \frac{1-\theta_w\beta}{1+\frac{\psi_l}{\eta}}\left(\tilde{U}_{2,n,t} - \tilde{U}_{1,n,t} + \tilde{P}_{n,t} + \frac{\Delta\tau^L_{n,t}}{1-\tau^L_n} + \frac{\Delta\tau^C_{n,t}}{1+\tau^C_n}\right) + \theta_w\beta\tilde{w}^*_{n,t+1}$$

Solving the reset equation for $\widetilde{w}^*_{n,t}$

$$(1-\theta_w)\widetilde{w}^*_{n,t} = \widetilde{W}_{n,t} - \theta_w\widetilde{W}_{n,t-1}$$

and substituting into the law of motion:

$$\widetilde{W}_{n,t} - \theta_w\widetilde{W}_{n,t-1} = \frac{(1-\theta_w)(1-\theta_w\beta)}{1+\frac{\psi_l}{\eta}}\left(\tilde{U}_{2,n,t} - \tilde{U}_{1,n,t} + \tilde{P}_{n,t} + \frac{\Delta\tau^L_{n,t}}{1-\tau^L_n} + \frac{\Delta\tau^C_{n,t}}{1+\tau^C_n}\right) + \theta_w\beta\left(\widetilde{W}_{n,t+1} - \theta_w\widetilde{W}_{n,t}\right)$$

Using $\widetilde{W}_{n,t} - \widetilde{W}_{n,t-1} = \tilde{\pi}^w_{n,t}$:

$$(1-\theta_w)\widetilde{W}_{n,t} + \theta_w\tilde{\pi}^w_{n,t} = \frac{(1-\theta_w)(1-\theta_w\beta)}{1+\frac{\psi_l}{\eta}}\left(\tilde{U}_{2,n,t} - \tilde{U}_{1,n,t} + \tilde{P}_{n,t} + \frac{\Delta\tau^L_{n,t}}{1-\tau^L_n} + \frac{\Delta\tau^C_{n,t}}{1+\tau^C_n}\right) + \theta_w\beta\left[(1-\theta_w)\widetilde{W}_{n,t} + \tilde{\pi}^w_{n,t+1}\right]$$
$$\theta_w\tilde{\pi}^w_{n,t} = \frac{(1-\theta_w)(1-\theta_w\beta)}{1+\frac{\psi_l}{\eta}}\left(\tilde{U}_{2,n,t} - \tilde{U}_{1,n,t} + \frac{\Delta\tau^L_{n,t}}{1-\tau^L_n} + \frac{\Delta\tau^C_{n,t}}{1+\tau^C_n} - \widetilde{w}_{n,t}\right) + \theta_w\beta\tilde{\pi}^w_{n,t+1}$$

3. Capital Euler equation[7]

$$(1+i_{n,t})F(\lambda_{n,t})e^{\epsilon^F_{n,t}} = \frac{\sum_{s^{t+1}} \pi(s^{t+1}|s_t)\left[(1-\tau^K_{n,t+1})u_{n,t+1}R_{n,t+1} + \mu_{n,t+1}\left(1-\delta(1-\tau^K_{n,t+1})\right) - P_{n,t+1}a\left(u_{n,t+1}\right))\right]}{\mu_{n,t}}$$

$$\frac{\beta}{F_n}\left((1-\tau^K_n)u_n r_n \tilde{r}_{n,t+1} - (u_n r_n - \delta)\Delta\tau^K_{n,t+1}\right) = \beta\Delta i_{n,t} - \tilde{\pi}_{n,t+1} + \frac{\Delta sp_{n,t}}{F_n} + \widetilde{\left(\frac{\mu_{n,t}}{P_{n,t}}\right)} - \frac{\beta}{F_n}(1-\delta(1-\tau^K_n))\widetilde{\left(\frac{\mu_{n,t+1}}{P_{n,t+1}}\right)}$$

[7] Log-linearizing gives

$$F_n\mu_n(1+i)\left(\tilde{\mu}_{n,t} + \frac{\Delta i_{n,t}}{1+i} + \frac{F'_n}{F_n}\lambda_n\tilde{\lambda}_{n,t} + \Delta\epsilon^F_{n,t}\right) = (1-\tau^K_n)u_n R_n\left(\tilde{u}_{n,t+1} + \tilde{R}_{n,t+1}\right) - (u_n R_n - \delta)\Delta\tau^K_{n,t+1}$$
$$+ (1-\delta(1-\tau^K_n))\mu_n\tilde{\mu}_{n,t+1} - a(u_n)P_n\tilde{P}_{n,t+1} - (1-\tau^K_n)R_n\tilde{u}_{n,t+1}$$

Simplifying:

$$\tilde{\mu}_{n,t} + \beta\Delta i_{n,t} + F_\epsilon\tilde{\lambda}_{n,t} + \Delta\epsilon^F_{n,t} = \frac{\beta}{F_n}\left((1-\tau^K_n)u_n r_n\tilde{R}_{n,t+1} - (u_n r_n - \delta)\Delta\tau^K_{n,t+1} + (1-\delta(1-\tau^K_n))\tilde{\mu}_{n,t+1} - a(u_n)\tilde{P}_{t+1}\right)$$

We replace $\Delta sp_{n,t}/F = F_\epsilon\tilde{\lambda}_{it} + \delta\epsilon^F_{n,t}$:

$$\widetilde{\left(\frac{\mu_{n,t}}{P_{n,t}}\right)} - \tilde{\pi}_{n,t+1} + \beta\Delta i_{n,t} + \Delta sp_{n,t}/F = \frac{\beta}{F_n}\left((1-\tau^K_n)u_n r_n\tilde{R}_{n,t+1} - (u_n r_n - \delta)\Delta\tau^K_{n,t+1} + (1-\delta(1-\tau^K_n))\tilde{\mu}_{n,t+1} - \left(\frac{F_n}{\beta} + a(u_n)\right)\tilde{P}_{n,t+1}\right)$$

Notice that $\frac{F(\lambda_n)}{\beta} + a\left(u_n\right) = (1-\tau^K_n)u_n r_n + \left(1-\delta(1-\tau^K_n)\right)$.

$$\beta\Delta i_{n,t} - \tilde{\pi}_{n,t+1} + \Delta sp_{n,t}/F + \widetilde{\left(\frac{\mu_{n,t}}{P_{n,t}}\right)} = \frac{\beta}{F_n}\left((1-\tau^K_n)u_n r_n\tilde{r}_{n,t+1} + (1-\delta(1-\tau^K_n))\widetilde{\left(\frac{\mu_{n,t+1}}{P_{n,t+1}}\right)} + (u_n r_n - \delta)\Delta\tau^K_{n,t+1}\right)$$

4. Price of capital[8]

$$\frac{U_{1,n,t}}{1+\tau^C_{n,t}} = \frac{\mu_{n,t}}{P_{n,t}}\frac{U_{1,n,t}}{1+\tau^C_{n,t}}\left(1-f-f'\frac{X_{n,t}}{X_{n,t-1}}\right) + \beta\sum_{s^{t+1}}\pi(s^{t+1}|s^t)\left[\frac{\mu_{n,t+1}}{P_{n,t+1}}\frac{U_{1,n,t+1}}{1+\tau^C_{n,t+1}}f'\left(\frac{X_{n,t+1}}{X_{n,t}}\right)^2\right]$$

$$\widetilde{\left(\frac{\mu_{n,t}}{P_{n,t}}\right)} = f''\left[(1+\beta)\widetilde{X}_{n,t} - \widetilde{X}_{n,t-1} - \beta\widetilde{X}_{n,t+1}\right]$$

5. Optimal capital utilization

$$(1-\tau^K_{n,t})r_{n,t} = a'(u_{n,t})$$

$$r_n\left(-\Delta\tau^K_{n,t} + (1-\tau^K_n)\tilde{r}_{n,t}\right) = a'' u_n \tilde{u}_{n,t}$$

6. Optimal factor employment

$$\frac{\alpha}{1-\alpha}\frac{w_{n,t}}{r_{n,t}} = \frac{u_{n,t}K_{n,t-1}}{L_{n,t}}$$

$$\tilde{r}_{n,t} - \widetilde{w}_{n,t} = \tilde{L}_{n,t} - \tilde{u}_{n,t} - \tilde{K}_{n,t-1}$$

7. Real marginal costs

$$MC_{n,t} = \frac{W^{1-\alpha}_{n,t}R^{\alpha}_{n,t}}{Z_{n,t}}\left(\frac{1}{1-\alpha}\right)^{1-\alpha}\left(\frac{1}{\alpha}\right)^{\alpha}$$

$$\widetilde{mc}_{n,t} = -\tilde{Z}_{n,t} + \alpha\tilde{r}_{n,t} + (1-\alpha)\widetilde{w}_{n,t}$$

---

[8]Recall the FOC:

$$C_{n,t}: \quad U_{1,n,t} = \lambda_{n,t}P_{n,t}(1+\tau^C_{n,t})$$

$$X_{n,t}: \quad \lambda_{n,t}P_{n,t} = v_{n,t}\left(1-f-f'\frac{X_{n,t}}{X_{n,t-1}}\right) + \beta\sum_{s^{t+1}}\pi(s^{t+1}|s^t)\left[v_{n,t+1}f'\left(\frac{X_{n,t+1}}{X_{n,t}}\right)^2\right]$$

$$K_{n,t+1}: \quad \lambda_{n,t}\mu_{n,t} - v_{n,t} = \beta\lambda_{n,t+1}(1-\delta)\mu_{n,t+1} - \beta v_{n,t+1}(1-\delta),$$

where $\lambda_{n,t}$ and $v_{n,t}$ are the multipliers on the budget constraint and the law of motion for capital. The last FOC implies that $v_{n,t} = \lambda_{n,t}\mu_{n,t}$. Inserting into the FOC for $X_{n,t}$ gives

$$\frac{U_{1,n,t}}{1+\tau^C_{n,t}} = \frac{\mu_{n,t}}{P_{n,t}}\frac{U_{1,n,t}}{1+\tau^C_{n,t}}\left(1-f-f'\frac{X_{n,t}}{X_{n,t-1}}\right) + \beta\sum_{s^{t+1}}\pi(s^{t+1}|s^t)\left[\frac{\mu_{n,t+1}}{P_{n,t+1}}\frac{U_{1,n,t+1}}{1+\tau^C_{n,t+1}}f'\left(\frac{X_{n,t+1}}{X_{n,t}}\right)^2\right]$$

8. FOC wrt $y^j_{n,t}$

$$y^j_{n,t} = Y_{n,t}\omega^j_n \left[\frac{E_{j,t}}{E_{n,t}}\frac{p_{j,t}}{P_{n,t}}\right]^{-\psi_y}$$

$$\widetilde{\left(\frac{p_{j,t}}{P_{j,t}}\right)} + \tilde{e}_{j,t} - \tilde{e}_{n,t} = \frac{1}{\psi_y}\left(\tilde{Y}_{n,t} - \tilde{y}^j_{n,t}\right) \quad \forall j$$

9. Production of $Q_{n,t}$

$$Q_{n,t} = Z_{n,t}\left(u_{n,t}K_{n,t-1}\right)^{\alpha} L^{1-\alpha}_{n,t}$$

$$\tilde{Q}_{n,t} = \tilde{Z}_{n,t} + \alpha\tilde{u}_{n,t} + \alpha\tilde{K}_{n,t-1} + (1-\alpha)\tilde{L}_{n,t}$$

10. Production of $Y_{n,t}$[9]

$$Y_{n,t} = \left(\sum_{j=1}^{N}\left(\omega^j_n\right)^{\frac{1}{\psi_y}}\left(y^j_{n,t}\right)^{\frac{\psi_y-1}{\psi_y}}\right)^{\frac{\psi_y}{\psi_y-1}}$$

$$\tilde{Y}_{n,t} = \sum_{j=1}^{N}\omega^j_n\tilde{y}^j_{n,t}$$

[9]Our calibration of the shares ω^j_n is $\omega^j_n = \frac{y^j_n}{Y_n}$, so that

$$Y^{\frac{\psi_y-1}{\psi_y}}\tilde{Y}_{n,t} = \sum_{j=1}^{N}\left(\omega^j_n\right)^{\frac{1}{\psi_y}} {y^j_n}^{\frac{\psi_y-1}{\psi_y}}\tilde{y}^j_{n,t}$$

can be simplified.

11. Market clearing for intermediate goods[10]

$$Q_{n,t} = \sum_{j=1}^{N} \frac{\mathbb{N}_j}{\mathbb{N}_n} y_{j,t}^n + \upsilon_n G_{n,t}$$

$$\frac{Q_n}{Y_n}\tilde{Q}_{n,t} = \sum_{j=1}^{N} \frac{\mathbb{N}_j Y_j}{\mathbb{N}_n Y_n} \omega_j^n \tilde{y}_{j,t}^n + \frac{\upsilon_n G_n}{Y_n}\tilde{G}_{n,t}$$

12. Market clearing for final goods[11]

$$Y_{n,t} = C_{n,t} + X_{n,t} + (1-\upsilon_n)G_{n,t} + a\left(u_{n,t}\right)K_{n,t}$$

$$Y_n\tilde{Y}_{n,t} = C_n\tilde{C}_{n,t} + X_n\tilde{X}_{n,t} + (1-\upsilon_n)G_n\tilde{G}_{n,t} + r_n(1-\tau_n^K)K_n\tilde{u}_{n,t} + a(u_n)K_n\tilde{K}_{n,t}$$

[10]Note that

$$Q_n\tilde{Q}_{n,t} = \sum_{j=1}^{N} \frac{\mathbb{N}_j}{\mathbb{N}_n} y_j^n \tilde{y}_{j,t}^n + \upsilon_n G_n \tilde{G}_{n,t}$$

[11]Note that

$$a(u_n) = u_n(1-\tau_n^K)r_n + 1 - \frac{F_n}{\beta} - \delta(1-\theta^K\tau_n^K)$$

and is zero if $u_n = 1$.

13. Phillips curve [12]

$$\theta_p\left(\tilde{\pi}_{n,t}+\widetilde{ToT}_{n,t}\right)=(1-\theta_p)(1-\theta_p\beta)\left[\widetilde{mc}_{n,t}-\widetilde{\left(\frac{p_{n,t}}{P_{n,t}}\right)}\right]+\theta_p\beta\left(\tilde{\pi}_{n,t+1}+\widetilde{ToT}_{n,t+1}\right)$$

14. Monetary Policy

[12]First, derive the log-linearized form of the reset equation:

$$\varphi^*_{n,t}=\frac{\psi_q}{\psi_q-1}\frac{\sum_{j=0}^{\infty}(\theta_p\beta)^j\sum_{s^{t+j}}\pi(s^{t+j}|s^t)\frac{C_{n,t+j}^{-\frac{1}{\sigma}}}{(1+\tau^C_{n,t+j})P_{n,t+j}}(p_{n,t+j})^{\psi_q}MC_{n,t+j}Q_{n,t+j}}{\sum_{j=0}^{\infty}(\theta_p\beta)^j\sum_{s^{t+j}}\pi(s^{t+j}|s^t)\frac{C_{n,t+j}^{-\frac{1}{\sigma}}}{(1+\tau^C_{n,t+j})P_{n,t+j}}(p_{n,t+j})^{\psi_q}Q_{n,t+j}}\equiv\frac{A_{n,t}}{B_{n,t}}.$$

Then, $\tilde{\varphi}_{n,t}=\tilde{A}_{n,t}-\tilde{B}_{n,t}$. Note that

$$A_{n,t}=\frac{\psi_q}{\psi_q-1}\frac{C_{n,t}^{-\frac{1}{\sigma}}}{(1+\tau^C_{n,t})P_{n,t}}p_{n,t}^{\psi_q}MC_{n,t}Q_{n,t}+\theta_p\beta E_tA_{t+1},$$

and similarly for $B_{n,t}$. Log-linearizing gives

$$\tilde{A}_{n,t}=(1-\theta_p\beta)\left(-\frac{1}{\sigma}\tilde{C}_{n,t}-\frac{\Delta\tau^C_{n,t}}{1+\tau^C_n}\tilde{P}_{n,t}+\psi_q\tilde{p}_{n,t}+\widetilde{MC}_{n,t}+\tilde{Q}_{n,t}\right)+\theta_p\beta E_t\tilde{A}_{n,t+1},$$

and similarly for $\tilde{B}_{n,t}$. It follows that

$$\tilde{\varphi}^*_{n,t}=(1-\theta_p\beta)\widetilde{MC}_{n,t}+\theta_p\beta\tilde{\varphi}^*_{n,t+1}.$$

Solving for $\tilde{\varphi}^*_{n,t}$

$$(1-\theta_p)\tilde{\varphi}^*_{n,t}=\tilde{p}_{n,t}-\theta_p\tilde{p}_{n,t-1}$$

Substituting into the law of motion

$$\tilde{p}_{n,t}=\theta_p\tilde{p}_{n,t-1}+(1-\theta_p)\tilde{\varphi}^*_{n,t}$$

gives

$$\tilde{p}_{n,t}-\theta_p\tilde{p}_{n,t-1}=(1-\theta_p)(1-\theta_p\beta)\widetilde{MC}_{n,t}+\theta_p\beta\left(\tilde{p}_{n,t+1}-\theta_p\tilde{p}_{n,t}\right)$$

Using $\tilde{p}_{n,t}-\tilde{p}_{n,t-1}=\tilde{\pi}_{n,t}+\widetilde{ToT}_{n,t}$:

$$(1-\theta_p)\tilde{p}_{n,t}+\theta_p\left(\tilde{\pi}_{n,t}+\widetilde{ToT}_{n,t}\right)=(1-\theta_p)(1-\theta_p\beta)\widetilde{mc}_{n,t}+(1-\theta_p)(1-\theta_p\beta)\tilde{P}_{n,t}+\theta_p\beta\left[(1-\theta_p)\tilde{p}_{n,t}+\left(\tilde{\pi}_{n,t+1}+\widetilde{ToT}_{n,t+1}\right)\right]$$

$$\theta_p\left(\tilde{\pi}_{n,t}+\widetilde{ToT}_{n,t}\right)=(1-\theta_p)(1-\theta_p\beta)\left[\widetilde{mc}_{n,t}-\widetilde{\left(\frac{p_{n,t}}{P_{n,t}}\right)}\right]+\theta_p\beta\left(\tilde{\pi}_{n,t+1}+\widetilde{ToT}_{n,t+1}\right)$$

- Floating exchange rate:

$$\Delta i_{n,t} = \phi_i \Delta i_{n,t-1} + (1-\phi_i)\left(\phi_Q \widetilde{Q}_{n,t} + \phi_\pi \tilde{\pi}_{n,t} + \epsilon^i_{n,t}\right)$$

- Fixed exchange rate:
  - Leader n:

$$\Delta i_{n,t} = \phi_i \Delta i_{n,t-1} + (1-\phi_i) \sum_{j \in CU} weight_j \left(\phi_Q \widetilde{Q}_{j,t} + \phi_\pi \tilde{\pi}_{j,t} + \epsilon^i_{n,t}\right),$$

    where $weight_j$ is the share of Q_j in the gdp of the currency union.
  - Follower j:

$$\widetilde{\Delta E}_{j,t} = \widetilde{\Delta E}_{n,t}$$

15. International Euler equation

- Complete markets

$$\tilde{U}_{1,n,t} = \tilde{e}_{n,t}$$

- Incomplete markets (Uncovered interest rate parity)

$$0 = \tilde{e}_{1,t}$$

$$\beta \Delta i_{n,t} - \tilde{\pi}_{n,t+1} + \tilde{e}_{n,t+1} - \tilde{e}_{n,t} = \beta \Delta i_{1,t} - \tilde{\pi}_{1,t+1} + \tilde{e}_{1,t+1} - \tilde{e}_{1,t} + \iota \frac{S^*_1}{Y_1} \tilde{S}^*_{1,t}. \qquad \text{for} \quad n > 1$$

16. Definition of change in nominal exchange rate

$$\widetilde{\Delta E}_{n,t} = (\tilde{e}_{n,t} - \tilde{e}_{n,t-1}) - \tilde{\pi}_{n,t}$$

17. Definition of Terms of Trade

$$\widetilde{ToT}_{n,t} = \widetilde{\left(\frac{p_{n,t}}{P_{n,t}}\right)} - \widetilde{\left(\frac{p_{n,t-1}}{P_{n,t-1}}\right)}$$

18. Definition of wage inflation

$$\tilde{\pi}^w_{n,t} = \tilde{\pi}_{n,t} + \widetilde{w}_{n,t} - \widetilde{w}_{n,t-1}$$

19. Law of motion for net worth of entrepreneurs [13]

$$NW_{n,t} = \frac{\beta}{F_n}\left\{K_{n,t-1}\left[(1-\tau^K_n)u_{n,t}R^k_{n,t} + \mu_{n,t}(1-\delta(1-\tau^K_n)) - P_{n,t}a\left(u_{n,t}\right)\right] - (1+i_{n,t-1})F_{n,t-1}B^e_{n,t-1}\right\}$$

$$\widetilde{NW}_{n,t} = \frac{\beta}{F_n}(1-\tau^K)r^k_n\lambda_n\tilde{r}^k_{n,t} + \frac{\beta}{F_n}(1-\delta(1-\tau^K_n))\lambda_n\widetilde{\left(\frac{\mu_{n,t}}{P_{n,t}}\right)} - (\lambda_n - 1)\left(\Delta i_{n,t-1} - \tilde{\pi}_t + \frac{\Delta sp_{n,t-1}}{F_n}\right) - \lambda_n\widetilde{\left(\frac{\mu_{n,t-1}}{P_{n,t-1}}\right)} + \widetilde{NW}_{n,t-1}$$

20. Leverage of entrepreneurs

$$\lambda_{n,t} = \frac{\mu_{n,t}K_{n,t+1}}{P_{n,t}NW_{n,t}}$$

$$\tilde{\lambda}_{n,t} = \widetilde{\left(\frac{\mu_{n,t}}{P_{n,t}}\right)} + \tilde{K}_{n,t+1} - \widetilde{NW}_{n,t}$$

[13] Dividing through by P_t gives

$$NW_{n,t}\frac{F_n}{\beta} = K_{n,t-1}\left[(1-\tau^K_n)u_{n,t}r^k_{n,t} + \frac{\mu_{n,t}}{P_{n,t}}(1-\delta(1-\tau^K_n)) - a\left(u_{n,t}\right)\right] - \frac{1+i_{n,t-1}}{\pi_{n,t}}F_{n,t-1}\left(\frac{\mu_{n,t-1}}{P_{n,t-1}}K_{n,t-1} - NW_{n,t-1}\right)$$

Log-linearizing gives

$$NW\frac{F_n}{\beta}\widetilde{NW}_{n,t} = \left(\frac{F_n}{\beta} - \frac{F_n}{\beta}\right)K_n\tilde{K}_{n,t-1} + (1-\tau^K)K_n r^k_n\tilde{r}^k_{n,t} + (1-\delta(1-\tau^K_n))K_n\widetilde{\left(\frac{\mu_{n,t}}{P_{n,t}}\right)} - (K - NW)\frac{F_n}{\beta}\left(\Delta i_{n,t-1} - \tilde{\pi}_t + \tilde{F}_{n,t-1}\right) - \frac{F_n}{\beta}K_n\widetilde{\left(\frac{\mu_{n,t-1}}{P_{n,t-1}}\right)} + \frac{F_n}{\beta}NW\widetilde{NW}_{n,t-1}$$

$$\widetilde{NW}_{n,t} = \frac{\beta}{F_n}(1-\tau^K)r^k_n\lambda_n\tilde{r}^k_{n,t} + \frac{\beta}{F_n}(1-\delta(1-\tau^K_n))\lambda_n\widetilde{\left(\frac{\mu_{n,t}}{P_{n,t}}\right)} - (\lambda_n - 1)\left(\Delta i_{n,t-1} - \tilde{\pi}_t + \tilde{F}_{n,t-1}\right) - \lambda_n\widetilde{\left(\frac{\mu_{n,t-1}}{P_{n,t-1}}\right)} + \widetilde{NW}_{n,t-1}$$

where we used that $u_n = 1$, $a'(1) = (1-\tau^K_n)r^k_n$, $\mu = P$, $\pi = 1$, $1+i = \frac{1}{\beta}$ and $(1-\tau^K_n)r^k_n + 1 - \delta(1-\tau^K_n) = \frac{F_n}{\beta}$ in steady state.

21. Definition of investment

$$\delta\tilde{X}_{n,t} = \tilde{K}_{n,t+1} - (1-\delta)\tilde{K}_{n,t}$$

22. Definition of interest rate spread

$$sp_{n,t} = F(\lambda_{n,t})e^{\epsilon^F_{n,t}} - 1$$
$$\Delta sp_{n,t} = F_n\left(F_\epsilon\tilde{\lambda}_{n,t} + \Delta\epsilon^F_{n,t}\right)$$

23. Marginal utility of consumption[14] With separable:

$$\widetilde{U}_{1,n,t} = -\frac{1}{\sigma}\tilde{c}_{n,t}$$

[14] **Separable preferences:**

$$U_{1,n,t} = c_{n,t}^{-\frac{1}{\sigma}}$$
$$\tilde{U}_{1,n,t} = -\frac{1}{\sigma}\tilde{c}_{n,t}$$

GHH preferences:

$$U_{n,t} = \frac{1}{1-\frac{1}{\sigma}}\left(c_{n,t} - \kappa_n\frac{L_{n,t}^{1+\frac{1}{\eta}}}{1+\frac{1}{\eta}}\right)^{1-\frac{1}{\sigma}}$$
$$(U_{1,n,t})^{-\sigma} = c_{n,t} - \kappa_n\frac{L_{n,t}^{1+\frac{1}{\eta}}}{1+\frac{1}{\eta}}$$

$$-\sigma\tilde{U}_{1,n,t} = \frac{C_n}{(U_{1,n})^{-\sigma}}\tilde{c}_{n,t} + \frac{L_n}{(U_{1,n})^{-\sigma}}\left(-\kappa_n L_n^{\frac{1}{\eta}}\right)\tilde{L}_{n,t}$$
$$-\sigma\tilde{U}_{1,n,t} = \frac{C_n}{(U_{1,n})^{-\sigma}}\tilde{c}_{n,t} - \frac{C_n}{(U_{1,n})^{-\sigma}}\frac{\kappa_n L_n^{1+\frac{1}{\eta}}}{Q_n}\frac{Q_n}{Y_n}\frac{Y_n}{C_n}\tilde{L}_{n,t}$$

Note that labor supply in steady state is

$$\kappa_n L_n^{\frac{1}{\eta}} = \frac{1-\tau^L_n}{1+\tau^C_n}\frac{W_n}{P_n} = \frac{1-\tau^L_n}{1+\tau^C_n}(1-\alpha)\frac{\psi_q - 1}{\psi_q}\frac{Q_n}{L_n},$$

so that

$$\frac{\kappa_n L_n^{1+\frac{1}{\eta}}}{Q_n} = \frac{1-\tau^L_n}{1+\tau^C_n}(1-\alpha)\frac{\psi_q - 1}{\psi_q}.$$

with GHH:

$$-\sigma\left(1-\frac{1-\alpha}{1+\frac{1}{\eta}}\frac{1-\tau_n^L}{1+\tau_n^C}\frac{\psi_q-1}{\psi_q}\frac{Q_n}{C_n}\right)\tilde{U}_{1,n,t} = \tilde{c}_{n,t}-(1-\alpha)\frac{1-\tau_n^L}{1+\tau_n^C}\frac{\psi_q-1}{\psi_q}\frac{Q_n}{C_n}\tilde{L}_{n,t}$$

with CD:

$$\widetilde{U}_{1,n,t} = \left(\left(1-\frac{1}{\sigma}\right)\kappa-1\right)\tilde{c}_{n,t} - \left(1-\frac{1}{\sigma}\right)\kappa(1-\alpha)\frac{1-\tau_n^L}{1+\tau_n^C}\frac{\psi_q-1}{\psi_q}\frac{Q_n}{C_n}\tilde{L}_{n,t}$$

Also:

$$\begin{aligned}
\frac{C_n}{(U_{1,n})^{-\sigma}} &= \frac{C_n}{C_n-\kappa_n\frac{L_n^{1+\frac{1}{\eta}}}{1+\frac{1}{\eta}}} \\
&= \left(1-\frac{1}{1+\frac{1}{\eta}}\frac{Y_n}{C_n}\frac{\kappa_n L_n^{1+\frac{1}{\eta}}}{Q_n}\frac{Q_n}{Y_n}\right)^{-1} \\
&= \left(1-\frac{1-\alpha}{1+\frac{1}{\eta}}\frac{1-\tau_n^L}{1+\tau_n^C}\frac{\psi_q-1}{\psi_q}\frac{Y_n}{C_n}\frac{Q_n}{Y_n}\right)^{-1}
\end{aligned}$$

Cobb-Douglas preferences:

$$U_{n,t} = \frac{\left(c_{n,t}^{\kappa}(1-L_{n,t})^{1-\kappa}\right)^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}}$$

$$\begin{aligned}
U_{1,n,t} &= \kappa c_{n,t}^{(1-\frac{1}{\sigma})\kappa-1}(1-L_{n,t})^{(1-\kappa)(1-\frac{1}{\sigma})} \\
\widetilde{U}_{1,n,t} &= \left[\left(1-\frac{1}{\sigma}\right)\kappa-1\right]\tilde{c}_{n,t}-\frac{L_n}{1-L_n}(1-\kappa)(1-\frac{1}{\sigma})\tilde{L}_{n,t}
\end{aligned}$$

Labor supply in steady state is

$$\begin{aligned}
\frac{1-\kappa}{\kappa}\frac{C_n}{L_n} &= (1-\alpha)\frac{1-\tau_n^L}{1+\tau_n^C}\frac{\psi_q-1}{\psi_q}\frac{Q_n}{L_n} \\
\frac{L_n}{1-L_n} &= \frac{\kappa}{1-\kappa}(1-\alpha)\frac{1-\tau_n^L}{1+\tau_n^C}\frac{\psi_q-1}{\psi_q}\frac{Q_n}{Y_n}\frac{Y_n}{C_n}
\end{aligned}$$

24. Marginal rate of substitution[15]

$$\tilde{U}_{2,n,t} - \tilde{U}_{1,n,t} = \frac{1}{\eta}\tilde{L}_{n,t} - \tilde{U}_{1,n,t}$$

With GHH:

$$\tilde{U}_{2,n,t} - \tilde{U}_{1,n,t} = \frac{1}{\eta}\tilde{L}_{n,t}$$

With CD:

$$\tilde{U}_{2,n,t} - \tilde{U}_{1,n,t} = \tilde{c}_{n,t} - \frac{\kappa}{1-\kappa}(1-\alpha)\frac{\psi_q - 1}{\psi_q}\frac{Q_n}{C_n}\tilde{L}_{n,t}$$

25. Hand-to-Mouth consumers[16]

$$C_{n,t} = (1-\chi)c_{n,t} + \chi m_n^{htm}\left(Y_{n,t} + \upsilon_n G_{n,t}\right)$$

$$\tilde{c}_{n,t} = \frac{1}{1-\chi}\widetilde{C}_{n,t} - \frac{\chi}{1-\chi}\frac{1}{Y_n + \upsilon_n G_n}\left(Y_n\tilde{Y}_{n,t} + \upsilon_n G_n\tilde{G}_{n,t}\right)$$

[15]**GHH preferences:**

$$U_{2,n,t} = -\kappa_n L_{n,t}^{\frac{1}{\eta}} U_{1,n,t}^h.$$

And log-linearizing gives

$$\tilde{U}_{2,n,t} - \tilde{U}_{1,n,t} = \frac{1}{\eta}\tilde{L}_{n,t}$$

Cobb-Douglas preferences:

$$U_{2,n,t} = -\frac{1-\kappa}{\kappa}\frac{c_{n,t}}{1-L_{n,t}}U_{1,n,t}.$$

And log-linearizing gives

$$\tilde{U}_{2,n,t} - \tilde{U}_{1,n,t} = \tilde{c}_{n,t} - \frac{L_n}{1-L_n}\tilde{L}_{n,t}$$

[16]We define hand-to-mouth consumption as

$$c_{n,t}^{htm} = m_n^{htm} Y_{n,t},$$

with $m_n^{htm} = C_n/Y_n$ in steady state.

26. Budget constraint (for incomplete market case)[17]

$$\frac{\Delta S^*_{1,t}}{Y_{1,t}} = 0$$

$$\tilde{Y}_{n,t} - \frac{Q_n}{Y_n}\left(\widetilde{\left(\frac{p_{n,t}}{P_{n,t}}\right)} + \widetilde{Q}_{n,t}\right) = \sum_{j\neq n}\frac{S^j_n}{Y_n}\left(\Delta i_{j,t-1} + \frac{1-\beta}{\beta}\left(\tilde{E}_{j,t} - \tilde{e}_{n,t}\right)\right) + \frac{1}{\beta}\frac{\Delta S^*_{n,t-1}}{Y_{n,t-1}} - \frac{\Delta S^*_{n,t}}{Y_{n,t}} \qquad \text{for} \quad n > 1$$

[17]Note that the quadratic penalty term on foreign bond holdings does not affect the log-linearized solution to the budget constraint. The full household budget constraint with incomplete markets is

$$P_{n,t}\left[C_{n,t} + X_{n,t}\right] + (1-\delta)\,\mu_{n,t}K_{n,t} + B_{n,t} + \frac{S^*_{n,t}}{E_{n,t}}$$
$$= \mu_{n,t}K_{n,t+1} + W_{n,t}L_{n,t} + \Pi^f_{n,t} + \Pi^e_{n,t} + (1+i_{t-1})F(\lambda_{n,t-1})e^{\epsilon^F_{n,t-1}}B_{n,t-1} + \frac{(1+i^*_{t-1})S^*_{n,t-1}}{E_{n,t}} - T_{n,t},$$

where $B_{n,t}$ are loans extended to domestic entrepreneurs. Use

$$T_{n,t} = G_{n,t}$$
$$C_{n,t} + X_{n,t} + G_{n,t} + a(u_{n,t})K_{n,t} = Y_{n,t}$$
$$W_{n,t}L_{n,t} + \Pi^f_{n,t} = p_{n,t}Q_{n,t} - R_{n,t}u_{n,t}K_{n,t}$$

to rewrite the budget constraint as

$$P_{n,t}Y_{n,t} - P_{n,t}a(u_{n,t})K_{n,t} + R_{n,t}u_{n,t}K_{n,t} + (1-\delta)\,\mu_{n,t}K_{n,t} - p_{n,t}Q_{n,t} - \frac{(1+i^*_{t-1})S^*_{n,t-1}}{E_{n,t}} + \frac{S^*_{n,t}}{E_{n,t}}$$
$$= \mu_{n,t}K_{n,t+1} + \Pi^e_{n,t} + (1+i_{t-1})F(\lambda_{n,t-1})e^{\epsilon^F_{n,t-1}}B_{n,t-1} - B_{n,t}.$$

For entrepreneurs, the budget constraint is

$$\mu_{n,t}K_{n,t+1} + \Pi^e_{n,t} + (1+i_{t-1})F(\lambda_{n,t-1})e^{\epsilon^F_{n,t-1}}B_{n,t-1} = R_{n,t}u_{n,t}K_{n,t} - P_{n,t}a(u_{n,t})K_{n,t} + (1-\delta)\mu_{n,t}K_{n,t} + B_{n,t}.$$

Inserting this into the households' budget constraint gives

$$P_{n,t}Y_{n,t} - p_{n,t}Q_{n,t} = \frac{(1+i^*_{t-1})S^*_{n,t-1}}{E_{n,t}} - \frac{S^*_{n,t}}{E_{n,t}}.$$

Collecting terms and dividing by $P_{n,t}$ gives

$$Y_{n,t} - \frac{p_{n,t}}{P_{n,t}}Q_{n,t} = \frac{(1+i^*_{t-1})S^*_{n,t-1} - S^*_{n,t}}{e_{n,t}},$$

which can be log-linearized to

$$\tilde{Y}_{n,t} - \frac{Q_n}{Y_n}\left(\widetilde{\left(\frac{p_{n,t}}{P_{n,t}}\right)} + \widetilde{Q}_{n,t}\right) = \frac{S^*_n}{Y_n}\Delta i^*_{t-1} + \frac{1}{\beta}\frac{\Delta S^*_{n,t-1}}{Y_{n,t-1}} - \frac{\Delta S^*_{n,t}}{Y_{n,t}} - \frac{S^*_n}{Y_n}\frac{1-\beta}{\beta}\tilde{e}_{n,t}.$$

Finally, we assume that net foreign asset positions are proportional to net export positions:

$$\tilde{Y}_{n,t} - \frac{Q_n}{Y_n}\left(\widetilde{\left(\frac{p_{n,t}}{P_{n,t}}\right)} + \widetilde{Q}_{n,t}\right) = \sum_{j\neq n}\frac{S^j_n}{Y_n}\left(\Delta i_{j,t-1} + \frac{1-\beta}{\beta}\left(\tilde{E}_{j,t} - \tilde{e}_{n,t}\right)\right) + \frac{1}{\beta}\frac{\Delta S^*_{n,t-1}}{Y_{n,t-1}} - \frac{\Delta S^*_{n,t}}{Y_{n,t}},$$

where we used the definition of $i^*_{t-1} = \sum_{j\neq n} E_{j,t}(1+i_{j,t-1})$.

3.2 Redundant Variables

1. Nominal net exports (in percent of steady-state GDP)

$$
\begin{aligned}
NX_{n,t} &= p_{n,t}\left(Q_{n,t} - \upsilon_n G_{n,t}\right) - P_{n,t} Y_{n,t} \\
\Delta NX_{n,t} &= (Q_n - \upsilon_n G_n)\tilde{p}_{n,t} + Q_n \widetilde{Q}_{n,t} - \upsilon_n G_n \tilde{G}_{n,t} - Y_n \left(\tilde{P}_{n,t} + \tilde{Y}_{n,t}\right) \\
\frac{\Delta NX_{n,t}}{Q_n} &= \left(1 - \frac{\upsilon_n G_n}{Q_n}\right)\tilde{p}_{n,t} + \widetilde{Q}_{n,t} - \frac{\upsilon_n G_n}{Q_n}\tilde{G}_{n,t} - \frac{Y_n}{Q_n}\left(\tilde{Y}_{n,t} - \tilde{P}_{n,t}\right)
\end{aligned}
$$

2. Change in real effective exchange rate

$$
\begin{aligned}
ee_{n,t} &= \sum_{j=1}^{N} sh_{n,j}\frac{e_n}{e_j} \\
\Delta\widetilde{ee}_{n,t} &= \Delta\tilde{e}_{n,t} - \sum_{j=1}^{N} sh_{n,j}\Delta\tilde{e}_{j,t}
\end{aligned}
$$

where $sh_{n,j} = \left(\frac{1}{2}\frac{\mathbb{N}_n y_n^j + \mathbb{N}_j y_j^n}{\mathbb{N}_n(Y_n + \upsilon_n G_n)}\right)$ is the average trade weight.

3. Change in nominal effective exchange rate

$$
\begin{aligned}
EE_{n,t} &= \sum_{j=1}^{N} sh_{n,j}\frac{E_n}{E_j} \\
\Delta\widetilde{EE}_{n,t} &= \Delta\tilde{E}_{n,t} - \sum_{j=1}^{N} sh_{n,j}\Delta\tilde{E}_{j,t}
\end{aligned}
$$

4. Price index of good purchased by government

$$
\tilde{P}^G_{n,t} = \upsilon_n\tilde{p}_{n,t} + (1 - \upsilon_n)\tilde{P}_{n,t}.
$$

5. Primary balance (in percent of steady-state GDP)[18]

$$
\begin{aligned}
PB_{n,t} &= \tau^C_{n,t} P_{n,t} C_{n,t} + \tau^L_{n,t} W_{n,t} L_{n,t} + \tau^K_{n,t} u_{n,t} R_{n,t} K_{n,t} - P^G_{n,t} G_{n,t} \\
\Delta PB_{n,t} &= \tau^C_n C_n \left(\tilde{\tau}^C_{n,t} + \tilde{P}_{n,t} + \tilde{C}_{n,t}\right) + \tau^L_n W_n L_n \left(\tilde{\tau}^L_{n,t} + \widetilde{W}_{n,t} + \tilde{L}_{n,t}\right) \\
&\quad + \tau^K_n K_n \left[u_n r_n \left(\tilde{\tau}^K_{n,t} + \widetilde{K}_{n,t}\right) + u_n r_n \left(\tilde{u}_{n,t} + \widetilde{R}_{n,t}\right)\right] - G_n \left(\tilde{P}^G_{n,t} + \widetilde{G}_{n,t}\right) \\
\frac{\Delta PB_{n,t}}{GDP_n} &= \tau^C_n \frac{C_n}{GDP_n} \left(\tilde{\tau}^C_{n,t} + \tilde{C}_{n,t}\right) + \tau^L_n \frac{W_n L_n}{GDP_n} \left(\tilde{\tau}^L_{n,t} + \widetilde{w}_{n,t} + \tilde{L}_{n,t}\right) - \frac{G_n}{GDP_n} \widetilde{G}_{n,t} \\
&\quad + \tau^K_n \frac{K_n}{GDP_n} \left[u_n r_n \left(\tilde{\tau}^K_{n,t} + \widetilde{K}_{n,t}\right) + u_n r_n \left(\tilde{u}_{n,t} + \tilde{r}_{n,t}\right)\right]
\end{aligned}
$$

6. Static primary balance (in percent of steady-state GDP)

$$
\frac{\Delta PB^{stat}_{n,t}}{GDP_n} = \tau^C_n \frac{C_n}{GDP_n} \tilde{\tau}^C_{n,t} + \tau^L_n \frac{W_n L_n}{GDP_n} \tilde{\tau}^L_{n,t} + \tau^K_n \frac{K_n}{GDP_n} u_n r_n \tilde{\tau}^K_{n,t} - \frac{G_n}{GDP_n} \widetilde{G}_{n,t}
$$

3.3 Combining Log-Linearized Equations

Production of the final good (10) Inserting the FOC wrt $y^j_{n,t}$

$$
\widetilde{\left(\frac{p_{j,t}}{P_{j,t}}\right)} + \tilde{e}_{j,t} - \tilde{e}_{n,t} = \frac{1}{\psi_y} \left(\tilde{Y}_{n,t} - \tilde{y}^j_{n,t}\right) \quad \forall j
$$

into the Production of the final good (10)

$$
\tilde{Y}_{n,t} = \sum_{j=1}^{N} \omega^j_n \tilde{y}^j_{n,t}
$$

gives

$$
0 = \sum_{j=1}^{N} \omega^j_n \left(\widetilde{\left(\frac{p_{j,t}}{P_{j,t}}\right)} + \tilde{e}_{j,t} - \tilde{e}_{n,t} \right)
$$

[18]We simplify by setting $PB_n = 0$, so that nominal price changes drop out.

Market clearing for intermediate good (11) Inserting the FOC wrt $y^n_{j,t}$

$$\widetilde{\left(\frac{p_{n,t}}{P_{n,t}}\right)} + \tilde{e}_{n,t} - \tilde{e}_{j,t} = \frac{1}{\psi_y}\left(\tilde{Y}_{j,t} - \tilde{y}^n_{j,t}\right) \quad \forall n$$

into the Market clearing for intermediate good (11)

$$\frac{Q_n}{Y_n}\tilde{Q}_{n,t} = \sum_{j=1}^{N} \frac{\mathbb{N}_j Y_j}{\mathbb{N}_n Y_n}\omega^n_j \tilde{y}^n_{j,t} + \frac{\upsilon_n G_n}{Y_n}\tilde{G}_{n,t}$$

gives

$$\frac{Q_n}{Y_n}\tilde{Q}_{n,t} - \frac{\upsilon_n G_n}{Y_n}\tilde{G}_{n,t} = \sum_{j=1}^{N} \frac{\mathbb{N}_j Y_j}{\mathbb{N}_n Y_n}\omega^n_j \left[\tilde{Y}_{j,t} - \psi_y\left(\widetilde{\left(\frac{p_{n,t}}{P_{n,t}}\right)} + \tilde{e}_{n,t} - \tilde{e}_{j,t}\right)\right]$$

Phillips curve (13) Inserting the Real marginal costs (7)

$$\widetilde{mc}_{i,t} = -\tilde{Z}_{i,t} + \alpha\tilde{r}_{i,t} + (1-\alpha)\widetilde{w}_{i,t}$$

and the Definition of Terms of Trade (17)

$$\widetilde{ToT}_{i,t} = \widetilde{\left(\frac{p_{i,t}}{P_{i,t}}\right)} - \widetilde{\left(\frac{p_{i,t-1}}{P_{i,t-1}}\right)}$$

into the Phillips curve (13)

$$\theta_p\left(\tilde{\pi}_{i,t} + \widetilde{ToT}_{i,t}\right) = (1-\theta_p)(1-\theta_p\beta)\left[\widetilde{mc}_{i,t} - \widetilde{\left(\frac{p_{i,t}}{P_{i,t}}\right)}\right] + \theta_p\beta\left(\tilde{\pi}_{i,t+1} + \widetilde{ToT}_{i,t+1}\right)$$

gives

$$\theta_p\left(\tilde{\pi}_{i,t} - \widetilde{\left(\frac{p_{i,t-1}}{P_{i,t-1}}\right)}\right) = (1-\theta_p)(1-\theta_p\beta)\left(-\tilde{Z}_{i,t} + \alpha\tilde{r}^k_{i,t} + (1-\alpha)\widetilde{w}^f_{i,t}\right) - (1+\theta_p^2\beta)\widetilde{\left(\frac{p_{i,t}}{P_{i,t}}\right)} + \theta_p\beta\left(\tilde{\pi}_{i,t+1} + \widetilde{\left(\frac{p_{i,t+1}}{P_{i,t+1}}\right)}\right)$$

Monetary policy (14) Inserting the Definition of change in nominal exchange rate (16)

$$\widetilde{\Delta E}_{i,t} = (\tilde{e}_{i,t} - \tilde{e}_{i,t-1}) - \tilde{\pi}_{i,t}$$

into the monetary policy rule for followers under fixed exchange rates (14)

$$\widetilde{\Delta E}_{j,t} = \widetilde{\Delta E}_{i,t}$$

gives

$$(\tilde{e}_{j,t} - \tilde{e}_{j,t-1}) - \tilde{\pi}_{j,t} = (\tilde{e}_{i,t} - \tilde{e}_{i,t-1}) - \tilde{\pi}_{i,t}$$

Wage Phillips curve (2) Inserting the Definition of wage inflation

$$\tilde{\pi}^w_{n,t} = \tilde{\pi}_{n,t} + \widetilde{w}_{n,t} - \widetilde{w}_{n,t-1}$$

into the Wage Phillips curve (2)

$$\theta_w \tilde{\pi}^w_t = (1-\theta_w)(1-\theta_w\beta)\left[\widetilde{U}_{2,n,t} - \widetilde{U}_{1,n,t} + \frac{\Delta\tau^L_{n,t}}{1-\tau^L_n} + \frac{\Delta\tau^C_{n,t}}{1+\tau^C_n} - \widetilde{w}_{n,t}\right] + \theta_w\beta\tilde{\pi}^w_{t+1}$$

gives

$$\theta_w\left(\tilde{\pi}_{n,t} - \widetilde{w}_{n,t-1}\right) = (1-\theta_w)(1-\theta_w\beta)\left[\widetilde{U}_{2,n,t} - \widetilde{U}_{1,n,t} + \frac{\Delta\tau^L_{n,t}}{1-\tau^L_n} + \frac{\Delta\tau^C_{n,t}}{1+\tau^C_n}\right] - \left(1+\theta_w^2\beta\right)\widetilde{w}_{n,t} + \theta_w\beta\left(\tilde{\pi}_{n,t+1} + \widetilde{w}_{n,t+1}\right)$$

Capital Euler equation (3) Inserting the Definition of the Leverage of entrepreneurs (20)

$$\widetilde{\lambda}_{n,t} = \widetilde{\left(\frac{\mu_{n,t}}{P_{n,t}}\right)} + \tilde{K}_{n,t} - \widetilde{NW}_{n,t}$$

into the Definition of interest rate spread (22)

$$\frac{\Delta sp_{n,t}}{F_n} = F_\epsilon\widetilde{\lambda}_{n,t} + \Delta\epsilon^F_{n,t}$$

gives

$$\frac{\Delta sp_{n,t}}{F_n} = F_\epsilon \left(\widetilde{\left(\frac{\mu_{n,t}}{P_{n,t}}\right)} + \tilde{K}_{n,t} - \widetilde{NW}_{n,t} \right) + \Delta\epsilon^F_{n,t}.$$

Inserting this into the Capital Euler equation (3)

$$\frac{\beta}{F_n}\left((1-\tau^K_n) r_n \tilde{r}_{n,t+1} - (r_n - \delta)\Delta\tau^K_{n,t+1}\right) = \beta\Delta i_{n,t} - \tilde{\pi}_{n,t+1} + \frac{\Delta sp_{n,t}}{F_n} + \widetilde{\left(\frac{\mu_{n,t}}{P_{n,t}}\right)} - \frac{\beta}{F_n}(1-\delta(1-\tau^K_n))\widetilde{\left(\frac{\mu_{n,t+1}}{P_{n,t+1}}\right)}$$

gives

$$\begin{aligned}\frac{\beta}{F_n}\left((1-\tau^K_n) r_n \tilde{r}_{n,t+1} - (r_n - \delta)\Delta\tau^K_{n,t+1}\right) &= \beta\Delta i_{n,t} - \tilde{\pi}_{n,t+1} + F_\epsilon \left(\widetilde{\left(\frac{\mu_{n,t}}{P_{n,t}}\right)} + \tilde{K}_{n,t} - \widetilde{NW}_{n,t} \right) + \Delta\epsilon^F_{n,t} \\ &\quad + \widetilde{\left(\frac{\mu_{n,t}}{P_{n,t}}\right)} - \frac{\beta}{F_n}(1-\delta(1-\tau^K_n))\widetilde{\left(\frac{\mu_{n,t+1}}{P_{n,t+1}}\right)}.\end{aligned}$$

Law of motion for net worth of entrepreneurs (19) Similarly, inserting the expression for the spread into the Law of motion for net worth of entrepreneurs (19)

$$\begin{aligned}\widetilde{NW}_{n,t} &= \frac{\beta}{F_n}(1-\tau^K) r^k_n \lambda_n \tilde{r}^k_{n,t} + \frac{\beta}{F_n}(1-\delta(1-\tau^K_n))\lambda_n \widetilde{\left(\frac{\mu_{n,t}}{P_{n,t}}\right)} - (\lambda_n - 1)\left(\Delta i_{n,t-1} - \tilde{\pi}_t + \frac{\Delta sp_{n,t-1}}{F_n}\right) \\ &\quad - \lambda_n \widetilde{\left(\frac{\mu_{n,t-1}}{P_{n,t-1}}\right)} + \widetilde{NW}_{n,t-1}\end{aligned}$$

gives

$$\begin{aligned}\widetilde{NW}_{n,t} &= \frac{\beta}{F_n}(1-\tau^K) r^k_n \lambda_n \tilde{r}^k_{n,t} + \frac{\beta}{F_n}(1-\delta(1-\tau^K_n))\lambda_n \widetilde{\left(\frac{\mu_{n,t}}{P_{n,t}}\right)} - (\lambda_n - 1)\left(\Delta i_{n,t-1} - \tilde{\pi}_t\right) \\ &\quad - (\lambda_n - 1)\left(F_\epsilon \left(\widetilde{\left(\frac{\mu_{n,t-1}}{P_{n,t-1}}\right)} + \tilde{K}_{n,t-1} - \widetilde{NW}_{n,t-1} \right) + \Delta\epsilon^F_{n,t-1}\right) - \lambda_n \widetilde{\left(\frac{\mu_{n,t-1}}{P_{n,t-1}}\right)} + \widetilde{NW}_{n,t-1}\end{aligned}$$